The CAPM and Beta in an Imperfect Market

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In recent work in this and another journal, Haim Levy develops the General Capital Asset Pricing Model (GCAPM), incorporating certain market imperfections (Levy, 1978 and 1980). He concludes that in GCAPM equilibrium, all investors do not necessarily hold the market portfolio and that a security's own variance is priced. The purpose of this note is to demonstrate that financial intermediaries, responding to potential abnormal profits, relax an important GCAPM constraint. The introduction of intermediaries into the GCAPM leads to results not unlike those of the CAPM itself. If an asset's own variance affects its price, we conclude that this feature provides a major reason for the existence of financial intermediaries.

Levy postulates that asset indivisibilities or transactions costs induce some investors to hold portfolios containing fewer than the N securities in the market portfolio. Since these portfolios are not fully diversified, they are not mean-variance efficient. Rather than plotting on the Capital Market Line (the ray rm in Figure 1), these portfolios lie on an interior efficient set, defined by the ray rk.

FIGURE 1
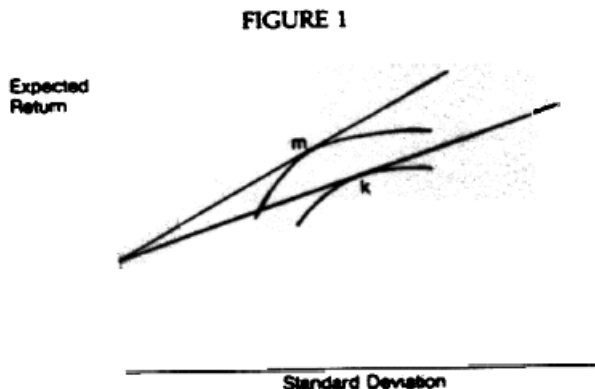
Since transactions costs and asset indivisibilities are proportionally less important for larger trades, the constraint described here is presumed to be less binding for wealthy investors. Therefore, the efficient sets of the wealthy lie closer to the Capital Market Line than do those of the less wealthy.

Under these conditions, investors of differing wealth select differing portfolios, even if expectations are homogeneous. Each holds a different interior portfolio and receives a different expected return per unit of risk borne. Moreover, since many investors hold only three or four stocks, Levy claims a security's own variance plays a "central role in equilibrium price determination" (Levy, 1978).

Sharpe (1978), on the other hand, notes that if residual risk were indeed priced, the investment behavior of financial intermediaries, less tightly restricted by wealth constraints, is clear: earn abnormal expected returns by simply constructing fully diversified portfolios.

Sharpe's argument, however, implies that persistent arbitrage opportunities exist, which is inconsistent with market efficiency.

A second point is more critical. The GCAPM requires investors with differing wealth constraints to receive different expected returns per unit of risk borne. This must mean that more than one price can exist in a market. Thus, the wealth constraint effectively separates the market for risk-bearing services, allowing price discrimination.

If this were the case, then we would expect arbitragers to pool the assets of many small investors and purchase large, well-diversified portfolios -- in principle. the market portfolio, m. Competition among arbitragers would insure that the gains are transferred to the small investors.

But this is precisely what we do observe! The arbitragers are financial intermediaries such as mutual funds and pension funds. To the extent that the costs of pooling, purchasing, and reissuing claims are larger than those paid by wealthy individuals, the funds would indeed plot below the Capital Market Line.

In addition, just as not all investors can afford to purchase m, not all can accumulate the minimum amount required to include the riskless asset in their portfolios. A wealth constraint exists for the riskless asset, as well as for risky assets.[1] Treasury bill funds, however, enable less-wealthy investors to earn the riskless rate, net of the intermediary's differential transactions costs. To the extent that these costs are uniform at all levels of portfolio risk, all investors' opportunity sets have the same slope.

Figure 2 illustrates equilibrium under these conditions. If expectations are homogeneous, all investors again hold the same portfolio of risky assets, m! Of course, they may well choose to bear a level of risk different from the risk that they would bear in a perfect market. The portfolio and leverage decisions are still separable, however.

FIGURE 2

Expected
Return

Standard Deviation
* denotes expected returns after transactions costs

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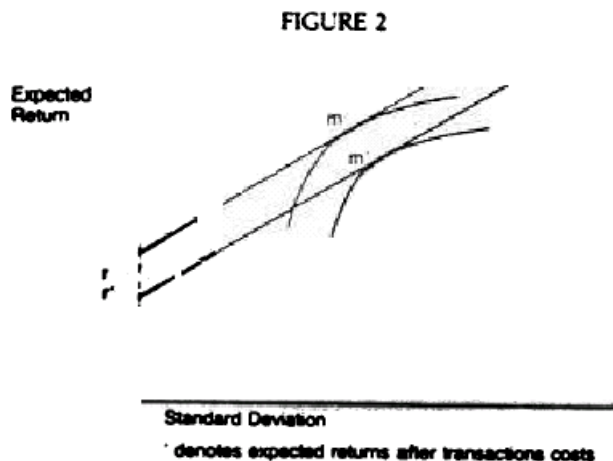
[1] Investors need somewhat less than $10,000 to purchase a Treasury bill, but unless they wish to bear no risk, they actually need more than this. They need $10,000 in addition to that invested in risky assets. Merton (1977) shows that with perfect markets and increasing, strictly concave utility functions, all investors hold at least one risky asset.

That all investors face precisely the same market for risk-bearing services is conditional on uniform costs of financial intermediation at all levels of portfolio risk. This assumption is not unrealistic, given the observed costs of intermediation for mutual funds.[2]

Since improvements in telecommunications and computer technology have reduced the costs of intermediation, we would expect the amount of institutional ownership and trading activity to increase over time. This, too, has been observed. For example, the New York Stock Exchange *Fact Book* reports estimated institutional holdings of stocks listed on the Exchange at five-year intervals.[3] In 1955, the first year for which data are available, the total value of NYSE stock held by institutions was $31.1 billion, representing 15% of total NYSE market value. Since then, both the dollar value and the percentage held by institutions has increased in every subsequent five-year period. As of 1980. the most recent year for which figures are available, institutions held $440.2 billion worth of NYSE stock, or 35% of total NYSE market value. At that time, more than 133 million individuals owned stock through intermediaries such as mutual and pension funds. The distribution of public trading volume on the NYSE was first reported for September 1952, and most recently for the fourth quarter of 1980. During that time. the percentage due to institutions more than doubled, from 31% to 65%.

In short, Levy is correct in noting that a wealth constraint may cause the opportunity sets of the wealthy to differ from those of the less wealthy. Our position is that financial intermediaries relax this constraint and insure that all investors can expect identical incremental returns for bearing an additional unit of risk. A corollary of the GCAPM, then, in addition to explaining how assets are priced, is that it provides an important reason why certain financial intermediaries exist: to invest individuals' funds. The result has been an investment process that has become increasingly institutionalized in recent decades.

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[2] For 1984, for example, the Vanguard Group reports a ratio of operating expenses to average net assets of 0.27% for its no-load stock portfolio, the Index Trust. For its Federal Portfolio, a no-load fund that invests in short-term government debt securities, the figure is 0.48%. This information is from the April 30, 1985 and March 31, 1985 prospectuses for the funds cited.

[3] All figures below are from the 1984 Fact Book.

Sharpe, William F. *Journal of Finance 33* (June 1978), pp. 917- 920. Discussion of Irwin Friend, Randolph Westerfield, and Michael Granito. "New Evidence on the Capital Asset Pricing Model," *Journal of Finance 33* (June 1978), pp. 903-917.